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Note to Professor Craig's Memoir, "Displacements Depending on One, Two and Three Parameters in a Space of Four Dimensions."

BY E. JAHNKE.

In volume XX of this Journal Professor Craig, generalizing the kinematical quantities p, q, r of ordinary space, has introduced six quantities $p_{ij}\,(i, j = 1, 2, 3, 4)$, as follows:

$$\begin{aligned} p_{12} &= -\sum \alpha_i \frac{d\beta_i}{dt}, \quad p_{13} = -\sum \gamma_i \frac{d\alpha_i}{dt}, \quad p_{23} = -\sum \beta_i \frac{d\gamma_i}{dt}, \\ p_{14} &= -\sum \alpha_i \frac{d\delta_i}{dt}, \quad p_{24} = -\sum \beta_i \frac{d\delta_i}{dt}, \quad p_{34} = -\sum \gamma_i \frac{d\delta_i}{dt}, \end{aligned} \qquad (i = 1, 2, 3, 4)$$

and established the following system of six differential equations analogous to the kinematical differential equations in the space of three dimensions:

$$\begin{aligned} \frac{d\alpha}{dt} &= p_{12}\beta - p_{13}\gamma + p_{14}\delta, \\ \frac{d\beta}{dt} &= -p_{12}\alpha + p_{23}\gamma + p_{24}\delta, \\ \frac{d\gamma}{dt} &= p_{13}\alpha - p_{23}\beta + p_{34}\delta, \\ \frac{d\delta}{dt} &= -p_{14}\alpha - p_{24}\beta - p_{34}\gamma. \end{aligned}$$

My memoir, "Ueber einen Zusammenhang zwischen den Elementen orthogonaler Neuner- und Sechzehnersysteme," Crelle's T., Bd. CXVIII, 224–233 appeared before. I have introduced there not only the six differential quantities

$$g\,p_{rs} = -\,(g_{1i}\,dg_{1j} + g_{2i}\,dg_{2j} + g_{3i}\,dg_{3j} + g_{4i}\,dg_{4j}),$$

but six other differential quantities

$$g\,v_{rs} = g_{i1}\,dg_{j1} + g_{i2}\,dg_{j2} + g_{i3}\,dg_{j3} + g_{i4}\,dg_{j4}, \quad \begin{aligned} i,\,j,\,r,\,s &= 1,\ 4,\ 2,\ 3 \\ &= 2,\ 4,\ 3,\ 1 \\ &= 3,\ 4,\ 1,\ 2 \end{aligned}$$

where

$$g = g_{1i}^2 + g_{2i}^2 + g_{3i}^2 + g_{4i}^2 = g_{i1}^2 + g_{i2}^2 + g_{i3}^2 + g_{i4}^2 .$$

I have obtained, with the aid of them, *two* systems of differential identities as follows:

$$g_{1i}\,d\log\frac{g_{1i}}{\sqrt{g}} = g_{1i}\,p_{rs} + g_{1r}\,p_{sj} + g_{1s}\,p_{jr} = -\,g_{2i}\,v_{34} - g_{3i}\,v_{42} - g_{4i}\,v_{23},$$

$$g_{2i}\,d\log\frac{g_{2i}}{\sqrt{g}} = g_{2j}\,p_{rs} + g_{2r}\,p_{sj} + g_{2s}\,p_{jr} = -\,g_{3i}\,v_{14} - g_{4i}\,v_{31} - g_{1i}\,v_{43},$$

$$g_{3i}\,d\log\frac{g_{3i}}{\sqrt{g}} = g_{3j}\,p_{r} + g_{3r}\,p_{sj} + g_{3s}\,p_{jr} = -\,g_{4i}\,v_{12} - g_{1i}\,v_{24} - g_{2i}\,v_{41},$$

$$g_{4i}\,d\log\frac{g_{4i}}{\sqrt{g}} = g_{4j}\,p_{rs} + g_{4r}\,p_{sj} + g_{4s}\,p_{jr} = -\,g_{1i}\,v_{32} - g_{2i}\,v_{13} - g_{3i}\,v_{21}.^{*}$$

Professor Craig's system is, with a little difference in notation, a special case of the first of my systems; it is obtained when we suppose $g = 1$.

I have employed my systems, it is true, for another purpose than Professor Craig's. I have deduced from them the general form which may be given to the differential equations of all problems referring to rotation. It is to be remarked that the introduction of the six differential quantities v_{rs} is necessary for the *complete* solution of dynamical problems. (Compare C. R. CXXVI, 1014 and Crelle's T., Bd. CXIX, 240.)

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[*] In my memoir is an erratum. Instead of dg_{mn} there must be read $g_{nm}\,d\log\frac{g_{mn}}{\sqrt{g}}$.